Contents

Preface **vii**

Main Notation **ix**

1 Variational aspects of some classes of elliptic problems **1**
1.1 Weak solvability results 2
1.2 Inhomogeneous boundary conditions 4
1.3 Elliptic systems 8
1.4 Necessary minimality conditions 14
1.5 Lower semicontinuity of integral functionals 16

2 Classical regularity theory for linear problems **29**
2.1 Caccioppoli-Leray inequality 29
2.2 Sobolev embeddings 34
2.3 A priori estimates and the Nirenberg method 37
2.4 Decay estimates for systems with constant coefficients . 43
2.5 Regularity up to the boundary 46

3 Interior regularity for nonlinear equations **51**
3.1 Hölder, Morrey and Campanato spaces 54
3.2 Hilbert's XIX problem and its solution in the two-dimensional case 61
3.3 Back to linear problems: Schauder theory 65
3.4 The space of BMO functions 70
3.5 Regularity in L^p spaces 78
3.6 De Giorgi's solution of Hilbert's XIX problem 87

4 Regularity for systems **99**
4.1 De Giorgi's counterexample to regularity for systems . . 99

4.2 Partial regularity for systems: basic definitions and ancillary results 104
4.3 Partial regularity for systems: $\mathscr{L}^n(\Sigma(u)) = 0$ 114
4.4 Partial regularity for systems: $\mathscr{H}^{n-2+\varepsilon}(\Sigma(u)) = 0$ . . . 118

5 Viscosity solutions 123
5.1 Basic definitions 123
5.2 Viscosity versus classical solutions 127
5.3 The distance function 130
5.4 Maximum principle for semiconvex functions 133
5.5 Existence and uniqueness results 139
5.6 The ABP maximum principle 143
5.7 A regularity result for viscosity solutions 156

A Some basic facts concerning Sobolev spaces 169
A.1 Two definitions and their comparison 169
A.2 Poincaré inequalities 175
A.3 Sobolev inequalities 179

B Miscellaneous results in real and harmonic analysis 185
B.1 Weak L^p spaces and the maximal operator 186
B.2 Some classical interpolation theorems 189
B.3 Lebesgue differentiation theorem 192
B.4 Calderón-Zygmund decomposition 193

C Hausdorff measures 197
C.1 Basic definitions 197
C.2 An ad hoc covering theorem 199
C.3 A comparison theorem for Hausdorff measures 201

D Some tools from convex and nonsmooth analysis 203
D.1 Subdifferential of a convex function 203
D.2 Convex functions and Measure Theory 210

References 213

Index 221