Preface

This text originated from the Ph.D. course given by the first author at the Scuola Normale Superiore in the academic year 2009-2010. The second and third authors, students of S.N.S. at that time, wrote down the first version of the notes. Since then the course has been replicated several times and the text has been gradually revised and expanded by the authors, until the present version.

All the material covered in this book is by now classical, the most recent contents being related to the theory of viscosity solutions. The presentation, the sequence of topics and even many proofs have been strongly inspired by the books by Giaquinta [41] and by Caffarelli-Cabré [15], the latter for the regularity theory of viscosity solutions. As in Evans' book [35], the guiding philosophy has been not to present the results in their maximal generality and under sharp technical assumptions, but rather to introduce the reader to the basic techniques of this beautiful and fundamental theory through the discussion of model cases. For this reason, this work is meant as a textbook and not as a reference book. Besides the treatises we already mentioned, the interested reader can consult many excellent and more systematic books on elliptic partial differential equations such as [44, 45, 49, 67, 83] among others, as well as the forthcoming monograph [74] on nonlinear Calderón-Zygmund theory. Further sources, more specific to the Calculus of Variations are, for instance, [24] and [47].

Ph.D. courses at S.N.S. are often attended also by students in the last years of their undergraduate studies. For this reason the course, while advanced in many respects, is meant to be self-contained, with four short appendices on Sobolev spaces, basic Real and Harmonic Analysis, Hausdorff measures and convex functions. Taking these appendices into account, only a basic knowledge of Functional Analysis and Measure Theory (and preferably also some fluency with Sobolev spaces of functions of one independent variable) is required.

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