

Contents

Introduction	i
1 Spontaneous symmetry breaking	1
1.1 Operational and mathematical description of a physical system	1
1.2 Symmetries and symmetry breaking	4
1.3 Infinitely extended systems. Pure phases and symmetry breaking	5
1.4 The Goldstone theorem	10
1.4.1 Examples	15
1.5 Goldstone theorem in relativistic local quantum field theory	19
2 Breaking gauge symmetries. Higgs mechanism	25
2.1 Global gauge symmetries	25
2.2 Local gauge symmetries	34
2.3 Local Gauss law	37
2.3.1 Gauss law, locality and gauge fixing	39
2.3.2 Gauss laws in Yang-Mills quantum theories	44
2.4 Higgs mechanism. Standard treatment	48
2.5 Non-perturbative constructive approaches to symmetry breaking	50
2.6 The Elitzur theorem	53
2.7 A non-perturbative argument for the absence of Goldstone bosons	55
2.8 A theorem on the Higgs phenomenon	58
2.9 Gauge symmetries, observables and states	66

3	The $U(1)$ problem in QCD	67
3.1	The $U(1)$ problem in QCD	67
3.1.1	The standard solution	69
3.1.2	A non-perturbative approach without instantons	76
3.2	Topology of the gauge group	77
3.2.1	The temporal gauge. The Gauss law and the physical states	77
3.2.2	The topology of the gauge group	80
3.2.3	The topological current	82
3.3	Gauge group topology solves the $U(1)$ problem	84
3.3.1	Fermions and chiral symmetry	84
3.3.2	Solution of the $U(1)$ problem	86
3.4	Gauge group topology and θ vacua	88
3.5	A prototypic model of QCD vacuum structure	94